Comparison of Diverse Optical CDMA Codes Using a Normalized Throughput Metric

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***Abstract*—A new method of comparing optical CDMA codes of different families, sizes and weights is described. We outline why the traditional performance metric of bit-error rate versus number of simultaneous users is lacking and propose a new performance measure—the peak throughput normalized with respect to the size of the code. This new metric is used to show that optical-orthogonal codes (OOCs) with a weight of 4 perform best at low offered loads while OOCs with weight 5 should be used at higher offered loads. By applying the technique across different families of codes, we demonstrate that multi-wavelength OOCs (MWOOCs) perform better than both OOCs (by a factor of approximately 1.25) and asymmetric prime-hop codes (by a factor of approximately 3.5), over a wide range of offered loads.**

***Index Terms*—Optical code-division multiple access (O-CDMA), optical fiber communication, optical fiber LAN.**

I. Introduction

OPTICAL code-division multiple-access (O-CDMA) is a method of sharing the bandwidth of optical fiber among a number of active users in a broadcast local-area network [1]. O-CDMA is asynchronous and operates without centralized control. O-CDMA is also flexible, allowing the spectral spreading to be tailored to the most appropriate domain: time, wavelength, or a combination of both.

The large number of diverse O-CDMA code families and possible sets of codewords within each family raises the question of which choice yields the best performance. In the O-CDMA literature, curves of bit-error rate (BER) versus the number of simultaneous users are typically used as the performance metric [2], [3]. However, it is very difficult to compare two sets of codewords based solely on the BER curves if the size and cardinality (maximum number of simultaneous users) are different. A code with a larger size should give better BER performance; however, it is important to consider how efficiently the extra chips are being used. In addition, a code with a large cardinality but relatively high BER may enable more aggregate data to be carried through a network than a code with good BER characteristics but a small cardinality.

Information-theoretic *capacity* has been proposed as an alternative metric for evaluating O-CDMA systems [4]. This fundamental quantity allows the performance of diverse networks to be distilled into a single number, making comparison straightforward. However, capacity represents an upper bound that may be very difficult to approach in practice. In addition, noise mechanisms with non-Gaussian distributions or the presence of a medium-access control (MAC) protocol may make the capacity difficult to calculate.

In this letter, we propose a new metric based on the network throughput normalized with respect to the size of the code. As will be shown below, this new metric has the qualities we desire in a figure of merit for optical CDMA networks. The code size and cardinality are both taken into account, and arbitrary BER and MAC models can be accommodated. We demonstrate the use of this normalized throughput metric by considering O-CDMA systems with pure time-domain (*one-dimensional*) codes [1] and wavelength-time (*two-dimensional*) codes [2].

II. Analytical Framework

We begin by modeling the O-CDMA system as a synchronous, random-access, packet broadcast network [5], [6]. Users start transmissions on common clock instances and the length of a slot corresponds to a packet of length L bits. Through the use of pseudo-orthogonal optical CDMA codes, a number of packets from different sources can be transmitted over the optical fiber in a single slot. We denote the number of simultaneous packets on the channel during a slot interval by m.

We assume that all sources of physical noise (such as shot, thermal and beat noise) can be neglected and include only the effects of crosstalk between users, termed multiple-access interference (MAI). This approximation is usually made in the literature on optical CDMA code design [1], [2]; however, in a power-limited regime of operation, the inclusion of physical noise may substantially change the throughput performance results shown in this letter.

Due to MAI, some of the packets will arrive at the receiver with bit errors. We let $P_B(m)$ be the probability of a bit error when there are m simultaneous transmissions on the channel. The form of $P_B(m)$ will depend upon the size, weight and family of the particular O-CDMA code under consideration. The probability of receiving a packet without errors when m simultaneous transmissions are on the channel is given by

$$P_C(m) = \begin{cases} [1 - P_B(m)]^L, & m \le \Phi \\ 0, & m > \Phi \end{cases} \tag{1}$$

where Φ is the cardinality of the code.

With suitable error-detection capability, the receiver can determine if one or more errors have occurred in a packet. All packets with errors are dropped by the receiver. For simplicity, we neglect the overhead required for this error-detection. In this

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broadcast network, the sender can independently determine the success or failure of the transmission and schedule the packet for retransmission after a random delay.

We let M be a random variable that represents the number of simultaneous transmissions in a time slot. The conditional distribution of the number of successfully received packets S is then

$$P[S=s|M=m] = \binom{m}{s} P_C^s(m)[1-P_C(m)]^{m-s}. \quad (2)$$

The steady-state throughput β can be shown [5] to equal

$$\beta = E[S] = E\left[E[S|M]\right] = \sum_{m=1}^{\infty} m P_C(m) f_M(m) \quad (3)$$

where $f_M(m)$ is the steady-state probability distribution of composite arrivals (new and retransmitted packets).

We assume that the composite arrival distribution is Poisson with arrival rate λ and write

$$f_M(m) = \frac{(\lambda T)^m}{m!} e^{-\lambda T} \quad (4)$$

where T is the temporal length of the packet. This choice of arrival distribution corresponds to an infinite user population. Defining $\gamma \triangleq \lambda T$ to be the offered load (average number of attempted transmissions per time slot), the throughput becomes [5]

$$\beta = e^{-\gamma} \sum_{m=1}^{\infty} m P_C(m) \frac{\gamma^m}{m!}. \quad (5)$$

A normalized throughput is defined according to

$$\beta' \triangleq \frac{\beta}{n_w n_t} \quad (6)$$

where n_w and n_t are the number of wavelengths and time chips, respectively, in the code. A one-dimensional code will have $n_w = 1$ while a two-dimensional code will have $n_w > 1$. The dependence of β' upon the offered load γ can be removed by determining the *peak* normalized throughput β'_{peak}. This is simply the largest value of β' attained over all possible values of γ.

Next, the values of β'_{peak} are plotted against the offered load at which the peak occurs γ_{peak} for many different code sets within the same family. The resulting curve suggests how the (normalized) throughput of the code family varies with changing network usage and provides a useful performance metric.

III. Optimal Weight for OOCs

To make the application of peak normalized throughput as a performance metric more concrete, we illustrate its use by determining the optimal weight for optical-orthogonal codes [1]. OOCs are a family of codes designed to have both a peak cross-correlation and a shifted autocorrelation equal to one. The choice of code weight w and length n_t for OOCs is arbitrary but these quantities both determine the cardinality according to

$$\Phi(n_t, w) \le \left\lfloor \frac{n_t - 1}{w(w-1)} \right\rfloor \quad (7)$$

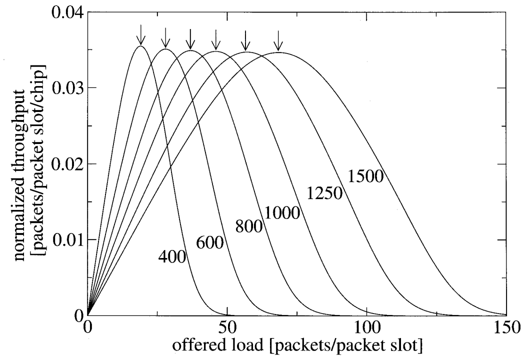

Fig. 1. Normalized throughput versus offered load for a family of optical orthogonal codes with weight equal to 4 and various lengths (indicated). The peak normalized throughput for each curve is indicated by an arrow.

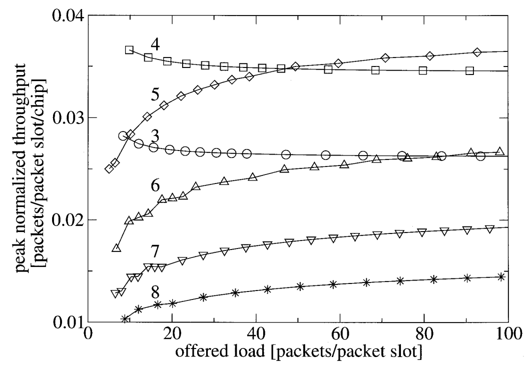

Fig. 2. Peak normalized throughput versus offered load for families of optical-orthogonal codes with various weights (indicated).

where the symbol $\lfloor z \rfloor$ denotes the integer portion of the real value z.

In Fig. 1, the normalized throughput is plotted as a function of the offered load for OOCs with weight $w = 4$ and various code lengths in the range $200 \le n_t \le 2250$ chips. The number of bits in a packet slot was fixed at $L = 1024$. The peak normalized throughput (indicated by the arrows in Fig. 1) varies by a relatively small amount as the length of the code is changed. Connecting the points corresponding to peak normalized throughput values yields a curve that is a function of offered load. This curve is plotted for codes of varying lengths with weight 4, as shown in Fig. 2. This figure also includes similar curves for OOCs with weights between 3 and 8.

The curves of peak normalized throughput versus the offered load indicate how the performance of a given code, with fixed weight, scales with increasing network usage. It is clear from Fig. 2 that OOCs with a weight $w \le 4$ show decreasing throughput as the offered load increases; those OOCs with a weight $w > 4$ show increasing throughput performance as the offered load rises. Fig. 2 also suggests that the optimum weight for an OOC at low offered loads ($\gamma < 48$) is $w = 4$ while at higher offered loads ($\gamma > 48$) a code with weight $w = 5$ should be chosen.

IV. Comparing Diverse Optical CDMA Codes

In the previous example, we considered only a single family of optical CDMA codes. However, the peak normalized

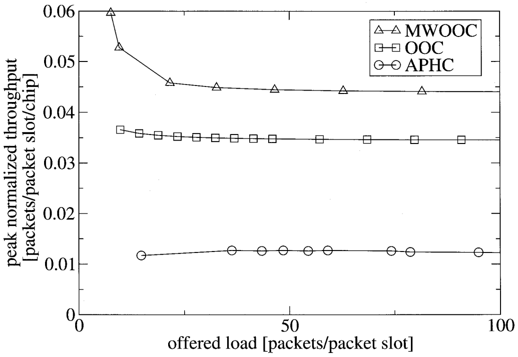

Fig. 3. Peak normalized throughput versus offered load for diverse optical CDMA codes. The OOC has weight $w = 4$ and the MWOOC has weight $w = 9$.

throughput metric is not constrained by code family and can be used to compare the performance of diverse optical CDMA codes. In this section, we consider three different optical CDMA codes: one-dimensional OOCs, two-dimensional multiple-wavelength optical-orthogonal codes (MWOOCs) and two-dimensional asymmetric prime-hop codes (APHCs). Using a similar technique as in Section III, we determine which of these three codes supports a system with the best throughput performance.

Multi-wavelength optical orthogonal codes [3] have a length in the time domain equal to the number of wavelengths used $(n_t = n_w = n)$, where n is a prime number. These codes are designed such that there is at most one pulse per wavelength. The shifted peak autocorrelation and cross-correlation are both equal to one. The cardinality of an MWOOC has an upper limit [3] of

$$\Phi(n \times n, w) \leq \left\lfloor \frac{n(n^2-1)}{w(w-1)} \right\rfloor. \quad (8)$$

Asymmetric prime-hop codes [2] have a length in the time domain of $n_t = p_s^2$, where p_s is a prime number. The number of wavelengths n_w must be both prime and larger than p_s. The weight is given by $w = p_s$. This family of codes has a cardinality [2] equal to

$$\Phi = p_s(n_w - 1). \quad (9)$$

In Fig. 3, we plot the peak normalized throughput versus offered load for these three codes. For all plots, the length of the packet slot was fixed at $L = 1024$ bits. We have chosen the OOC with weight $w = 4$ as the representative for the optical-orthogonal code family. The lengths of OOCs in the time domain were chosen in the range $200 \leq n_t \leq 2250$ chips. The procedure in Section III was repeated for the MWOOC to determine which weight yielded the best throughput performance. This optimal MWOOC, with weight $w = 9$, was chosen as the representative for this family of optical CDMA codes. The MWOOCs in Fig. 3 had a length the time domain of $11 \leq n \leq 47$ chips. For the APHCs, the weight is not a free parameter, but depends upon the degree of spreading in the time domain. The APHC curve in Fig. 3 represents the data from a large number of different APHCs with a length in the time domain in the range $49 \leq n_t \leq 841$ chips and number of wavelengths in the range $19 \leq n_w \leq 29$. The fact that the APHC curve is smooth even though the weight, size and aspect ratio of the code vary to a wide degree suggests that the peak normalized throughput is a good measure of the effectiveness of a code family.

Fig. 3 demonstrates that of the three code families under consideration, the MWOOC has the best performance. At high offered loads, the MWOOC outperforms the OOC by a factor of $\sim$ 1.25 and the APHC by a factor of $\sim$ 3.5. The fact that the one-dimensional OOC can perform better than the two-dimensional APHC is surprising. The extra degree of freedom afforded by the wavelength domain should allow multiple-access interference to be better controlled in two-dimensional codes. This is demonstrated by the superior performance of the MWOOCs compared to the OOCs.

V. Conclusion

The peak normalized throughput was demonstrated to be a useful tool in the design of O-CDMA networks for selecting specific code parameters, such as family and weight. By changing the BER mechanism to account for physical noise, or allowing a different medium-access control layer to be used, this metric is flexible enough to model any realistic O-CDMA local-area network system. Further research is necessary to determine how these additional parameters would change the throughput performance curves presented in this letter.

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