Optical CDMA Using 2-D Codes: The Optimal Single-User Detector

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***Abstract*—We consider the error performance of an optical code-division multiple access network in which two-dimensional codes are generated in time and wavelength. We show from first principles that the optimum single-user detection scheme which yields is the AND detector. By replacing the widely considered SUM detector with the AND detector, the channel capacity can be at least doubled for a given data rate, number of active users, and bit error rate. We have also shown that the error performance of a random code gives a tight upper bound on the performance of deterministic code with the same weight and dimension.**

***Index Terms*—Optical CDMA, optical communications, maximum-likelihood detector.**

I. Introduction

OPTICAL CDMA (O-CDMA) has attracted significant recent attention with a view to enabling high-capacity multiaccess local-area networking [1]. Two-dimensional O-CDMA, in which codes are constructed on multiple wavelengths as well as many time chips [2], [3], has gained increased attention with the maturation of wavelength-division multiplexed fiber-optic technology.

Decisions at the receiver of such systems are most often made by thresholding the total received energy after correlation with the desired code. While this method benefits from simplicity, it has not been shown to be optimal, even among single-user detection schemes, with respect to minimizing the error-inducing effects of multiple-access interference. In response, we derive from first principles in the present work the optimum single-user detector for multi-wavelength O-CDMA (MW-O-CDMA) systems for an arbitrary coding scheme. We seek to find a scheme which will minimize bit error rate and, therefore, also maximize efficiency of use of the channel for a given allowed error rate. Our focus on improving efficiency is motivated by widespread findings in previous works on O-CDMA which have typically resulted in efficiencies of only a few percent. We focus on the single-user case in order to arrive at a detection scheme which can be implemented at optical rates.

We assume a star topology in an intensity modulation/direct detection (IM/DD) system. We assume chip synchronous network traffic—the worst-case scenario with respect to error performance. The number of wavelengths used in the code is denoted by N_w, while the number of time chips used in one bit period per wavelength is denoted by L_t. The total number of chips available across the coding area is $N_w L_t$. The weight W of the code is the number of ones in the code, i.e., the number of chip periods during which an optical intensity pulse is present. We assume that all codes have the same weight.

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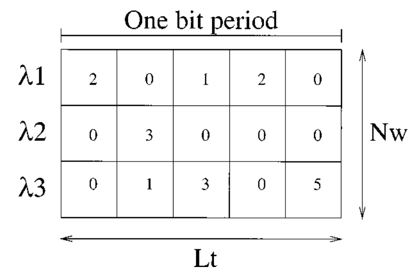

Fig. 1. The received signal in a 2-D optical CDMA system.

Since our system is a multiple access network, the signal received by any one of the users is a superposition of a subset of the codes transmitted by all of the active transmitters. An example of this is shown in Fig. 1. In Fig. 1, the number shown in each chip (square) represents the intensity of light detected at each chip by the receiver.

Since each chip is orthogonal to any other chip, the dimension D of the signal is $N_w L_t$. Any signal received by any user can be represented by the vector

$$\vec{r} = [r_1, r_2, \ldots, r_D]$$

where each component represents the intensity of the light received at the corresponding chip. As the signal power increases, multiple-access interference (MAI) becomes the BER-limiting mechanism. We, therefore, restrict our attention in the present analysis to the influence of MAI on error performance.

II. Detection Method

When bit "1" ($\vec{s_1}$) and bit "0" ($\vec{s_0}$) are equiprobable, the *maximum-likelihood* (ML) criterion for detection will minimize the average probability of error. In on–off keying, the corresponding decision rule is

$$P(\vec{r}|\vec{s_1}) \underset{\vec{s_0}}{\overset{\vec{s_1}}{\gtrless}} P(\vec{r}|\vec{s_0}). \tag{1}$$

If we assume that the received light intensity at any chip is independent of that at any other chip, the probability in (1) can be expressed as

$$P(\vec{r}|\vec{s_m}) = \prod_{k=1}^{D} P(r_k|s_{mk}), \qquad m = 0, 1. \tag{2}$$

This assumption is valid if either of the following cases is satisfied:

1) The codes used in the system are randomly generated with a uniform distribution.
2) In case of well-designed deterministic codes, the receiver does not take into account the algorithmic constructions of other users' codes in the system. Then, the receiver can only assume the presence of a pulse at a certain chip does not affect the probability of the presence of a pulse at any other chip.

Under the above conditions, the probability that a pulse from any other active user falling on a particular chip is $W/2D$. The detected intensity at a chip indicates the number of other users overlapping at this chip. Since the user transmitting a bit "0" does not send any pulse, the component likelihood functions when a bit "0" was sent are

$$P(r_k|s_{0k}) = \binom{N_{su}-1}{r_k}\left(\frac{W}{2D}\right)^{r_k}\left(1-\frac{W}{2D}\right)^{N_{su}-1-r_k}, \quad k = 1, 2, \ldots, D \tag{3}$$

where N_{su} is the number of active users in the network. Similarly, when a bit "1" was sent, the component likelihood functions are

$$P(r_j|s_{1j}) = \binom{N_{su}-1}{r_j-1}\left(\frac{W}{2D}\right)^{r_j-1}\left(1-\frac{W}{2D}\right)^{N_{su}-r_j}, \quad j = 1, 2, \ldots, W \tag{4}$$

where j denotes the "1" chip locations. Substituting (3) and (4) into (2), the decision rule finally becomes

$$\prod_{j=1}^{W}\left(\frac{N_{su}-r_j}{r_j}\right) \underset{\vec{s_0}}{\overset{\vec{s_1}}{\lesseqgtr}} \prod_{j=1}^{W}\left(\frac{2D}{W}-1\right). \tag{5}$$

The largest value of the left-hand side of (5) is infinity when there is a 0 in at least one of the r_j locations. In this case, the decision is $\vec{s_0}$. The second largest value of the left-hand side occurs when all the r_j are 1. In this case, the decision rule becomes

$$(N_{su}-1)^W \underset{\vec{s_0}}{\overset{\vec{s_1}}{\lesseqgtr}} \left(\frac{2D}{W}-1\right)^W. \tag{6}$$

In case of deterministic codes, N_{su} cannot exceed $(2D/W - 1)$ since $(2D/W - 1)$ is larger than the cardinality of any deterministic MW-O-CDMA code. Hence, the decision made from (5) is $\vec{s_1}$ as long as the light intensity levels of all the "1" chips are at least 1. This is equivalent to carrying out a logical AND operation on all the "1" chips. N_{su} is not involved in the process. Although the cardinality for the random codes can exceed $(2D/W-1)$ and hence threshold adjustment is required as N_{su} grows, we have shown that involving N_{su} in making decision typically improves the BER by only a small amount. The MAP detector for the MW-O-CDMA system using OOK is therefore, for nearly all practical cases, a logical AND operator.

The AND detector structure is equivalent to the correlation receiver with optical hard-limiter [4] first proposed by Salehi [5] in 1989. Similar results have also been shown for the special case of 1-dimensional optical orthogonal codes (OOC) [4], [6]–[8]. However, we have shown that the optical AND gate receiver structure is the optimal single-user detector for the MW-O-CDMA system if the code properties and physical noises are not taken into account.

In multi-wavelength O-CDMA systems proposed in literature [2], [3], the detection method suggested is to threshold the sum of the light intensities detected at all "1" chips. Individual light intensities at each "1" chip are not taken into account in making a decision. In the following sections, the conventional detector is referred as the SUM detector while the one described by (5) is called the AND detector.

III. Performance Analysis

In OOK, since each symbol only represents one bit, the symbol error rate is equal to the bit error rate. If $\vec{s_1}$ and $\vec{s_0}$ are equiprobable, the BER is

$$\text{BER} = \tfrac{1}{2}\,P(e|\vec{s_1}) + \tfrac{1}{2}\,P(e|\vec{s_0}). \tag{7}$$

For the AND detector, (5) is the decision rule. When an $\vec{s_1}$ was sent, light intensity detected at each of the W "1" chips must be at least 1. Thus, $P(e|\vec{s_1}) = 0$ in (7). The second term accounts for the probability that pulses sent by other users cause every "1" chip to have a light intensity of at least 1. Hence, (7) becomes

$$\text{BER} = \frac{1}{2}\prod_{j=1}^{W}\sum_{i=1}^{N_{su}-1}\binom{N_{su}-1}{i}\left(\frac{W}{2D}\right)^i\left(1-\frac{W}{2D}\right)^{N_{su}-1-i}. \tag{8}$$

For the SUM detector, the decision rule is

$$\sum_{j=1}^{W} r_j \underset{\vec{s_0}}{\overset{\vec{s_1}}{\gtrless}} W. \tag{9}$$

When a bit "1" was sent, the sum of the light intensities received at the W "1" chips must be at least W. Hence, $P(e|\vec{s_1}) = 0$ in (7). The BER for this conventional detector simplifies to the probability that at least W pulses from the $N_{su}-1$ users fall on the W "1" chips. The BER for the SUM detector is therefore

$$\text{BER} = \frac{1}{2}\sum_{i=W}^{W(N_{su}-1)}\binom{W(N_{su}-1)}{i}\left(\frac{W}{2D}\right)^i \cdot \left(1-\frac{W}{2D}\right)^{W(N_{su}-1)-i}. \tag{10}$$

The BER performance for the two detectors is compared in Fig. 2. Fig. 2 shows that the BER performance of the AND detector is always better than that of the SUM detector. Therefore, the code dimension of the system using the AND detector may be decreased so that its BER performance matches that using the SUM detector. Consequently, the spectral efficiency of this system, as defined in [9], is increased.

Fig. 3 shows the spectral efficiency improvement factor for different code weights. Both the values of N_{su} and D affect exactly how much improvement the AND detector can give over the SUM detector. The values depicted in Fig. 3 lead to less than

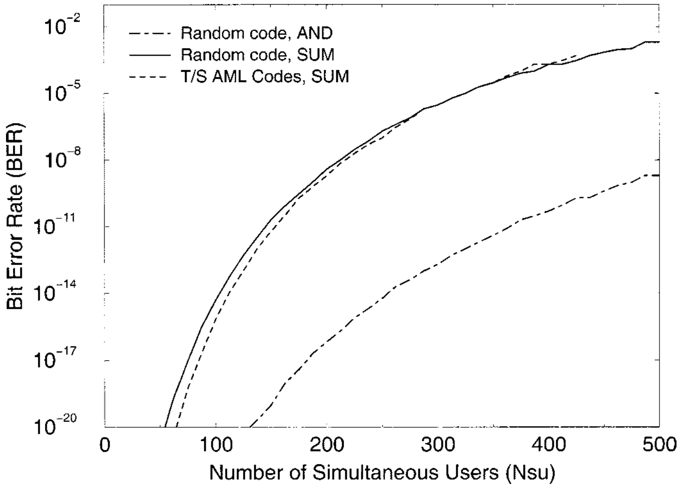

Fig. 2. BER performance against the number of active users (N_{su}). For all 3 curves, $N_w = 23$, $L_t = 435$ and $W = 23$.

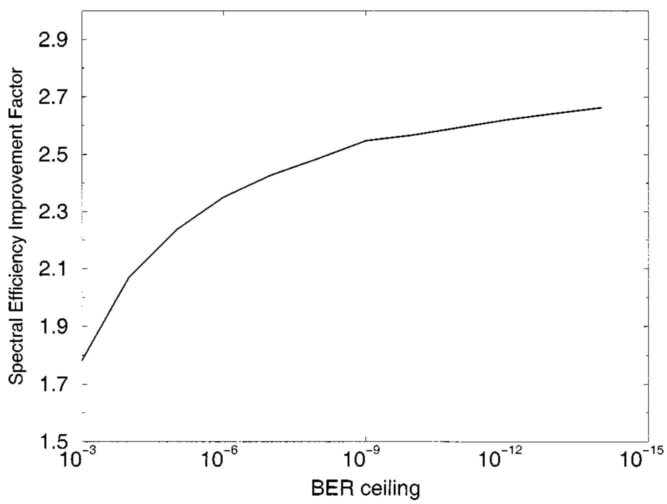

Fig. 3. Spectral efficiency improvement factor resulted from the AND detector for different code weights.

one order of magnitude of BER difference between the AND detector and the SUM detector for a large range of N_{su} (10 to $2D/W$) and D (1000 to 10 000). Hence, the code weight plays the most important role in determining how much spectral efficiency improvement the AND detector can give over the SUM detector. Fig. 3 shows that the benefit of the AND detector becomes more significant as the code weight is increased.

The BER performance of the T/S AML code [2] with the same parameters using the SUM detector is also shown in Fig. 2. Since the expression used to plot this curve includes the unity cross-correlation property of the code, it achieves a better BER than the random code for small N_{su}. However, as the number of active users increases, MAI quickly offsets the low cross-correlation advantage of the well-designed codes. The BER performance of a deterministic code asymptotically approaches that of the corresponding random code as N_{su} increases. The BER of a random code with the same weight and dimension therefore serves as a close ceiling estimate for that of the deterministic code.

IV. Conclusions

We have shown that the AND detector is the optimum single-user detector which gives the lowest averaged BER for a two-dimensional O-CDMA code. The decision is bit "1" only if the threshold is exceeded in all of the "1" chips. By replacing the conventional SUM detector with the AND detector, the spectral efficiency can be at least doubled with the same bandwidth, number of active users, and BER.

We have further shown that the BER performance of a random code can serve as a tight upper bound on that of any deterministic code with the same weight and dimension. In respect of multiple access interference, therefore, the AND detector is the optimum single-user detector for any code with any dimension and weight.

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