Nonlinear Disordered Media for Broad-Band Optical Limiting

Lukasz Brzozowski and Edward H. Sargent

***Abstract*—We analyze broad-band limiting behavior in nonlinear structures that are, on average, periodic. Introduction of a controlled degree of randomness in layer thicknesses results in widening of the stopband. Light at all of the frequencies in this broadened effective stopband of the structure with randomly perturbed layer thicknesses exhibits true optical limiting—clamping of the transmitted intensity below a fixed level independent of incident intensity. We explore the impact of the degree of randomization and the strength of the nonlinear mechanism on the smoothness and regularity of the limiting spectrum and on the localization of the light within these cyclostationary media. Optical limiting in integrable devices is of interest in optical logic, signal processing, and personnel and sensor protection.**

***Index Terms*—Author, please supply index terms. E-mail keywords@ieee.org for info.**

I. INTRODUCTION

PROPAGATION of light through media, which are on average periodic, but which exhibit some disorder, has been studied extensively in recent years [1]–[11]. Localization of photons, analogous with Anderson localization of electrons, has received special attention [1], [2], [7], [8].

The introduction of optical nonlinearity into such systems dramatically alters their response [9]–[11]. In the linear case, the intensity is described by exponential decay. The localization (characteristic) length is related to the average period and the transmittance of a unit cell [7], [11]. In the case of a nonlinear medium, the decay is much slower. The intensity envelope decays not faster than $1/x^2$, or even as $1/x$ for the case of strong nonlinearity and disorder present [9]–[11].

In this paper, we explore the propagation of light in disordered nonlinear media in the context of a particularly promising and important application: broad-band optical limiting.

We have previously presented a method for constructing all-optical limiters based on perfectly ordered nonlinear periodic structures [12]–[14]. These optically stable devices exhibit true optical limiting by clamping the transmitted intensity to a sensor-safe level. We have argued that our reflection-based approach should not suffer from such limitations as vulnerability to material damage or loss of the coherence of the transmitted signal. These problems occur in absorption-, self-defocusing-, or photorefractive-based mechanisms [15]–[17]. Extension of our approach to three-dimensionally periodic photonic crystals will result in angularly independent optical limiting.

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In this paper, we investigate how the introduction of disorder widens the effective stopband of the structure. Increasing disorder produces transmittance spikes within the effective bandgap—a phenomenon associated with the creation of defect states inside an otherwise ordered medium. Such behavior is undesirable in broad-band optical limiters.

The principle of superposition that applies in linear structures no longer applies in nonlinear media. Extending structure length will, therefore, not address the problem of local transmittance maxima. The old maxima may vanish but new maxima will appear. In order to counteract this problem, we propose fabricating composite structures in which a number of shorter structures are separated by absorbing optical isolators, which eliminate coupling between adjacent units via backward-propagating waves. Since each unit is, in general, different, it will have transmittance maxima at different frequency. For a large number of units, no transmittance maxima are present.

We study the localization of light within a number of structures. We observe light trapping with increasing incident intensity. We also explore the dependence of the strength of localization on the degree of disorder.

II. THEORETICAL MODEL

The structures analyzed are shown in Fig. 1. They consist of alternating layers of materials, each one possessing a Kerr nonlinearity. The index of refraction of a Kerr materials is expressed as

$$n = n_0 + n_{\mathrm{nl}} I \tag{1}$$

where

- n_0 linear part;
- n_{nl} Kerr coefficient;
- I local intensity of light in the medium.

The coefficient n_{nl} can be either positive or negative [18]. Thus, depending on the sign of n_{nl}, the index of refraction of the given material can either increase or decrease as the intensity is increased. We analyze structures whose average layer thicknesses are chosen to achieve the quarter-wave condition for the average index of refraction of 1.5 at a frequency of 5.64×10^{14} Hz ($\lambda_0 = 0.532$ μm). The thicknesses of the individual layers d_i are allowed to vary uniformly from their average value $d_{\lambda/4}$ within a specified range δ

$$d_i = d_{\lambda/4} \pm \delta. \tag{2}$$

In order to obtain an analytical expression for the evolution of forward- and backward-propagating field envelopes inside the

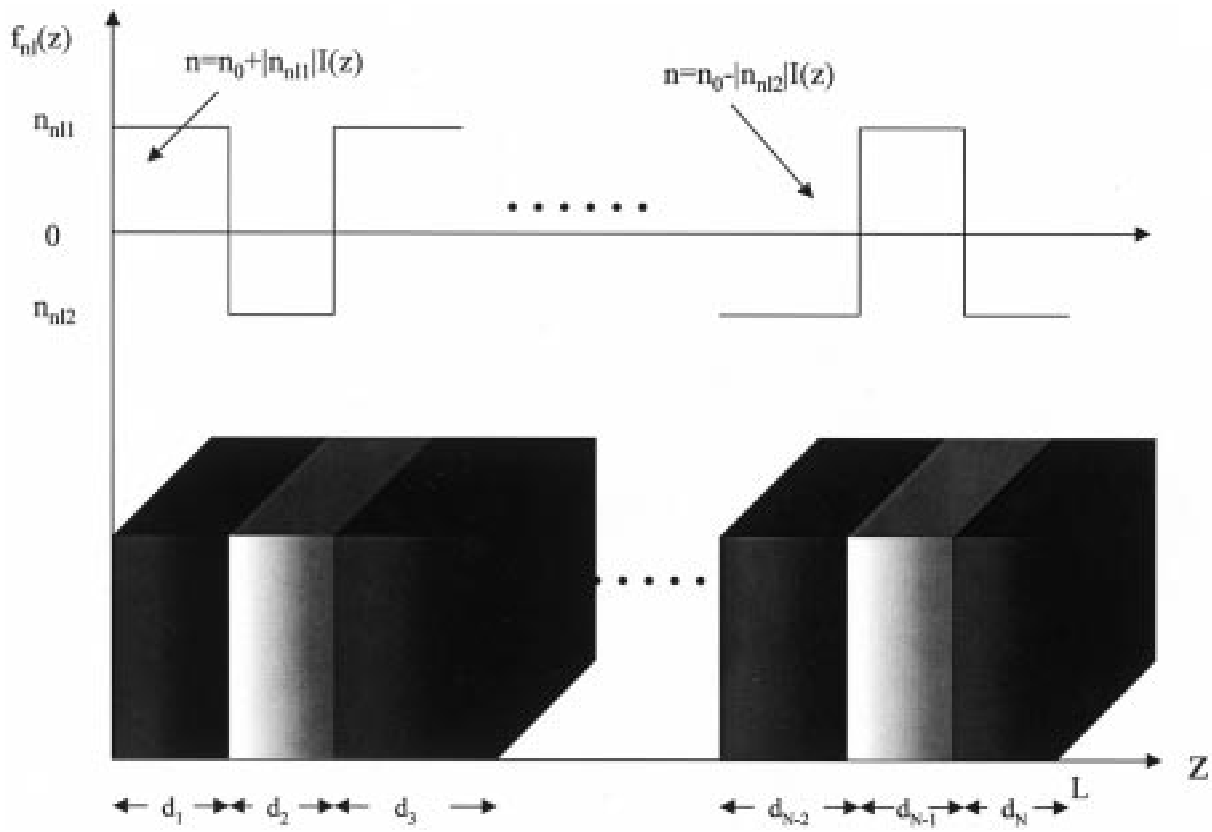


Fig. 1. Structures that are periodic on average and in which the magnitude of the Kerr nonlinearity is constant throughout and alternates in sign from layer to layer.

structure, we express the electric field at position z within the structure as

$$E(z) = A_1(z)e^{i(\omega n_0/c)z} + A_2(z)e^{-i(\omega n_0/c)z} \tag{3}$$

where

- A_1 and A_2 the coefficients of the forward- and backward-propagating waves, respectively;
- ω frequency of the radiation;
- c speed of light in vacuum.

We then modify the coupled mode formalism [19] for the case of a nonlinear periodic medium. We obtain coupled mode equations describing index-matching of the two materials

$$i\,\frac{dA_1(z)}{dz} = \frac{\omega}{c}\left[f_{nl}(z)I(z)A_2(z)e^{i(2\omega n_0/c)z} - \overline{n}_{\rm nl}I(z)A_1(z)\right] \tag{4}$$

$$i\,\frac{dA_2(z)}{dz} = -\frac{\omega}{c}\left[f_{\rm nl}(z)I(z)A_1(z)e^{-i(2\omega n_0/c)z} - \overline{n}_{\rm nl}I(z)A_2(z)\right] \tag{5}$$

where $I(z) = |A_1(z)|^2 + |A_2(z)|^2$, $f_{\rm nl}(z)$ specifies the Kerr coefficient at z (Fig. 1), and α is the absorption coefficient. In order to counteract multistability [14], we have restricted our analysis to the case of structures in which, in adjacent layers, Kerr coefficients are of the same magnitude but opposite sign. $\overline{n}_{\rm nl}$ is the average nonlinear index computed individually for each simulation according to the expression

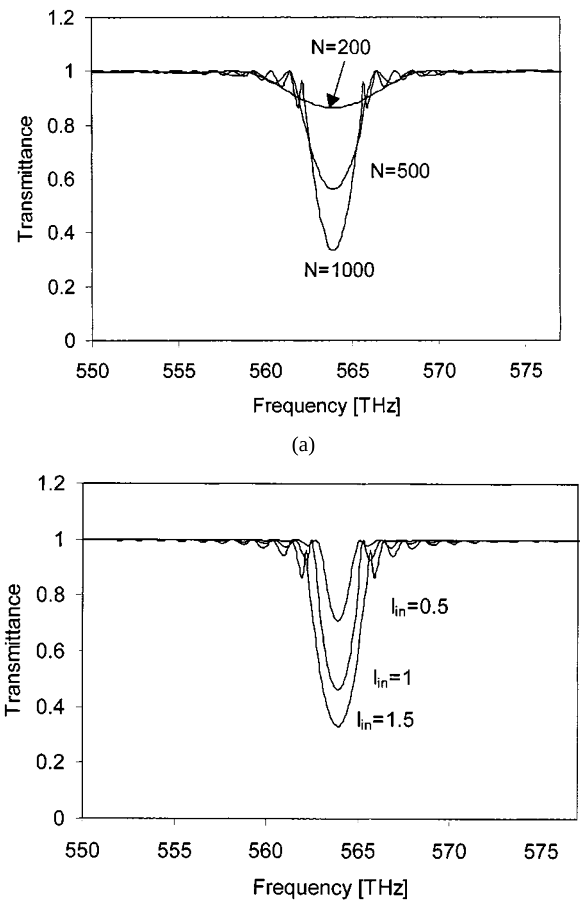
$$\overline{n}_{\rm nl} = \frac{1}{L}\sum_{i=1}^{N} n_{\rm nli}\, d_i \tag{6}$$

where L is the length of the structure. We have employed the slowly varying envelope approximation [19] in obtaining (4) and (5). Two boundary conditions were specified, both at position $z = L$: $A_2(L) = 0$, i.e., no radiation is incident on the structure from the right and $A(L) = A_{1\rm out}$, the assumed transmitted field. We take $A_{1\rm out}$ to be a real number. This is justified by invariance of the system under the global gauge transformation $(E\{z\} \rightarrow e^{i\theta}E(z))$ [20].

III. Results and Discussion

A. Spectral Analysis

We show in Fig. 2 the transmittance spectra of the perfectly ordered structures. The symmetric stopband remains fixed at the desired frequency. This behavior is a consequence of our use of materials with Kerr coefficients of identical magnitude but opposite sign. Throughout our analysis, we refer to the center angular frequency of the ordered structure as ω_0. As the number of layers is increased, the stopband grows sharper and deeper [Fig. 2(a)]. In Fig. 2(b), we show that increasing incident intensity widens the stopband. This can be understood via analogy to the linear structures where the width of the stopband is proportional to the index contrast [19]. Since, in the structures analyzed, Kerr indexes of the two materials used are of opposite



(b)

Fig. 2. Transmittance spectra of the ordered structures. The structures analyzed have linear index of refraction 1.5 and ($n_{\mathrm{nl1}} = -n_{\mathrm{nl2}} = 0.002$). (a) Impact of number of layers for incident intensity fixed at 1.5. (b) Impact of incident intensity for number of layers fixed at 1000.

sign, increasing incident intensity increases the nonlinear index contrast.

We have provided an in-depth physical analysis of the ordered multilayer limiting structures elsewhere [12], [13]. In this work, we concentrate instead on imperfectly ordered systems.

We show in Fig. 3 the effect of introducing disorder. The structures analyzed consisted of 1000 layers and had a linear index of 1.5, and the magnitude of the Kerr indexes $|n_{\mathrm{nl}}| = 0.003$. We have taken the system to be illuminated with normalized intensity of one (units reciprocal to those of n_{nl}). The individual layer thicknesses were distributed uniformly within 1%, 7%, and 10% of the quarter-wave value. The spectral width on which a given structure acts increases with the degree of disorder. For a 1% deviation, the transmittance spectrum is very similar to the unperturbed response. For a 7% percent deviation, most of the limiting strength is still concentrated close to the center of the unperturbed stopband. Increasing the degree of randomness to 10% spreads the effective stopband over a much wider range.

Two distinct features are quite general.

1) Increased disorder, which widens the stopband, degrades the depth of the stopband. Randomly varying layer thickness decreases the availability of wavevectors for which the Bragg condition is satisfied at a given optical frequency. A larger range of frequencies experiences some degree of backscattering. The strength of this backscattering is proportional to the number of coherent scatterers and the proximity of the individual layers to the quarter-wave value. A randomized system with a fixed number of layers will, therefore, have a smaller number of strongly backscattering regions than the ordered one.
2) The introduction of randomness makes it possible for a particular frequency of light at a specific intensity to see a high transmittance even if it lies within the new, wider effective stopband (Fig. 3). For a randomized system, there is a possibility of obtaining a phase difference of close to 2π between the incident and reflected waves. In this case, constructive interference between forward- and backward-propagating waves results in a transmittance spike. Such behavior—associated with photonic defect states—is not be observed within the stopband of an ordered structure made up of materials with opposite Kerr coefficients.

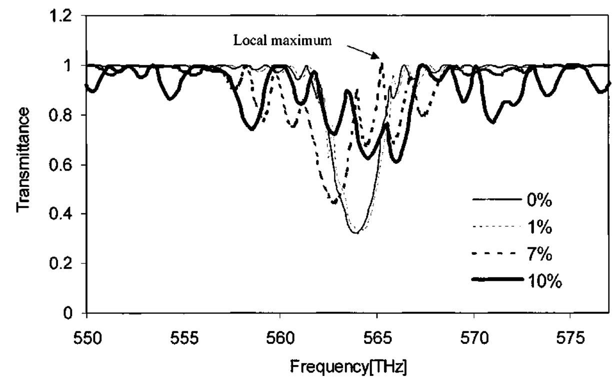


Fig. 3. Influence of increased layer thickness randomization on nonlinear transmittance spectra.

The detailed transmittance spectra of structures with randomly varying layer thicknesses depend on the details of random thickness fluctuations. It is not sufficient to specify statistical properties of the structure and materials: the details of a particular random trial will determine the detailed spectrum and, in particular, the location of any transmittance maxima within the former stopband. We illustrate this effect in Fig. 3. The more the given structures are randomized, the less closely their responses resemble one another from trial to trial. We shall exploit this fact to design a broad-band optical limiter with no transmittance maxima within the effective stopband.

In Fig. 4, we propose inserting optical isolators, which absorb only in the reverse (leftward) direction, between sections of limiters. Because the principle of superposition does not apply in nonlinear structures—thus, n structures each made up of N layers differ in their response from a single (nN)-layer stucture—combining the limiters without the isolators will not eliminate local transmittance maxima within the stopband. The maxima of the individual units may disappear from the transmittance spectra, but new features originating from the combined nonlinear distributed feedback structure will appear.

We plot in Fig. 5 the transmittance spectra of a limiter made up of increasing numbers of 1000-layer units with 10% randomness, $n_0 = 1.5$, and $|n_{\mathrm{nl}}| = 0.002$, illuminated with an inten-

Fig. 4. A combined system consisting of broad-band optical limiters coupled using absorbing optical isolators. Decoupling among the constituent limiters eliminates transmittance maxima within the effective stopband.

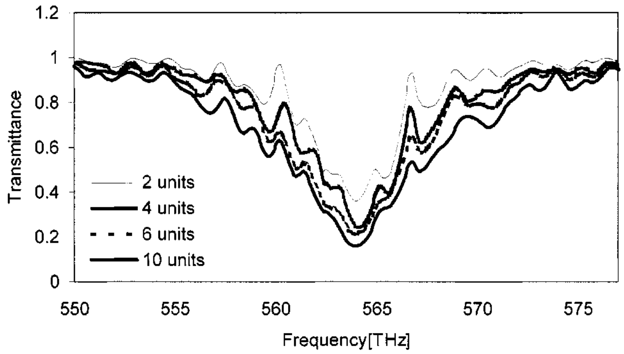


Fig. 5. Combining randomized units in series eliminates transmittance maxima and deepens and widens the band.

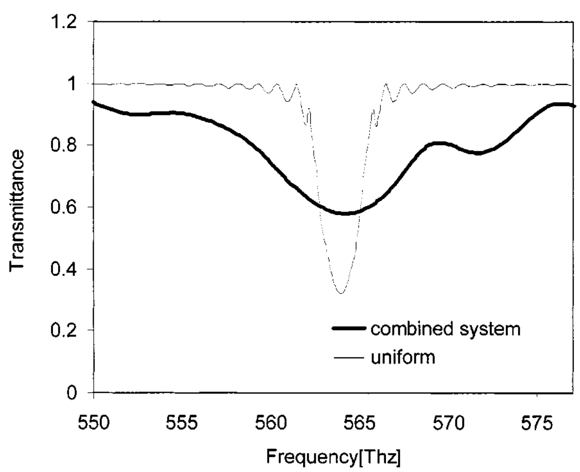


Fig. 6. Comparison of transmission spectra for a single 1000-layer structure (thin line) versus five combined randomized 200-layer units (thick line).

sity of three. The 1000-layer units are separated by optical isolators. In the many-unit system with isolators, the transmittance maxima that are present in the shorter systems are eliminated. A wide stopband with no undesired transmittance maxima is formed.

We compare in Fig. 6 the transmittance spectra of two 1000-layer structures: a combined system of five 7% randomized 200-layer units and a 1000-layer ordered system. The system with isolators acts on a much wider range of frequencies.

B. Intensity Analysis

We now demonstrate that the randomized systems exhibit true limiting behavior: they clamp the transmitted intensity below a fixed level, which does not depend on incident intensity. This behavior is highly desirable in sensor and personnel protection and in strongly nonlinear signal-processing operations, including all-optical logic [13].

We show in Fig. 7(a) the transmitted versus incident intensity for a structure of 100 layers with $|n_{\rm nl}| = 0.01$ for various degrees of randomness. The frequency of the optical signal lies at the center of the stopband. All of the structures exhibit saturation to a limiting intensity. The stronger the randomization, the higher the limiting intensity. However, true limiting behavior is preserved even in the presence of a high degree of randomness. As we have shown previously in [12], [13], the choice of Kerr indexes of opposite sign and comparable magnitude is essential in order to preserve this limiting behavior. The structure would otherwise exhibit multistability.

In order for the broad-band limiter to be effective, it must display limiting behavior over its entire stopband. Since the width of the stopband is proportional to the index contrast, and in the structures analyzed, index contrast is proportional to $I_{\rm in}$, it is necessary to fix the incident intensity in comparing the broad-band characteristics of ordered and randomized structures.

We display in Fig. 7(b) the transmitted versus incident intensity for a 100-layer, $|n_{\rm nl}| = 0.01$, 10% randomized structure at various frequencies. Even without optical isolators, the structures behave like limiters. However, unless a structure employing optical isolators is used, there is a possibility that light at a particular frequency will only see limiting at a much higher intensity than the rest of the effective stopband (560 THz in Fig. 7). Such behavior manifests itself with resonance spikes present in the transmittance spectra for a range of $I_{\rm in}$. Introducing isolators ensures that the structures exhibit limiting behavior over the entire stopband (i.e., no resonance spikes), and at intermediate frequencies.

We compare in Fig. 8 the transmittance ($T = I_{\rm out}/I_{\rm in}$) as a function of incident intensity for the structure of Fig. 7(b) and an ordered 100-layer unit. Near ω_0 (565 THz), ordered and disordered structures start to display limiting properties at comparable incident intensity. For the off-center frequencies (585 THz), randomized structures begin to display limiting behavior at lower intensities. This confirms that, at a particular $I_{\rm in}$ the effective broadband is larger for disordered structures.

C. Localization of Light

We show in Fig. 9 the evolution of the intensity associated with the forward-propagating wave across a 100-layer structure with $|n_{\rm nl}| = 0.01$, illuminated with an intensity of two. These structures involve no isolators. As the degree of randomization is increased, the forward-propagating wave experiences weaker attenuation—its localization length increases and the limiting strength decreases. For high degrees of disorder (20% and 30%), light exhibits localization.

We illustrate in Fig. 10 the localization of light within the 30% randomized structure for various values of incident intensity. As the incident intensity is increased, a nonlinear grating is formed and light becomes trapped within the structure. However, as

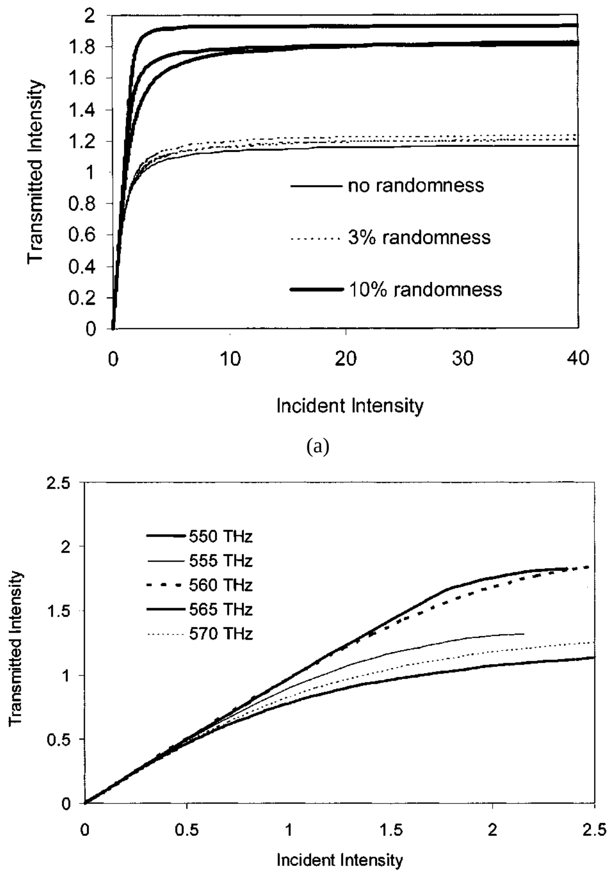


Fig. 7. (a) Transmitted versus incident intensity for various degrees of randomness at various trials. (b) Transmitted versus incident intensity for frequencies over the broadened stopband of a 10% randomized 100-layer structure.

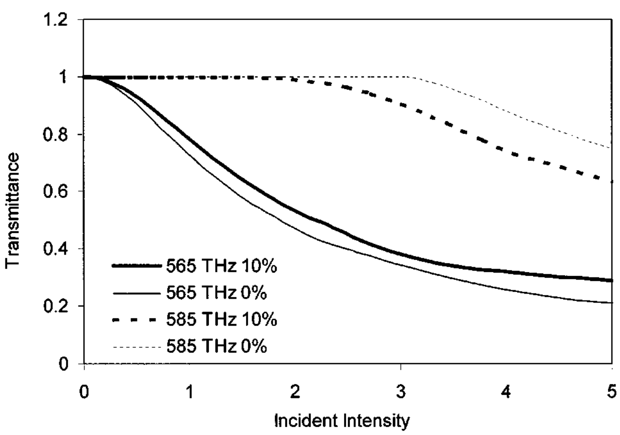


Fig. 8. Transmittance versus incident intensity. Comparison between ordered and 10% randomized 100-layer structures at two frequencies.

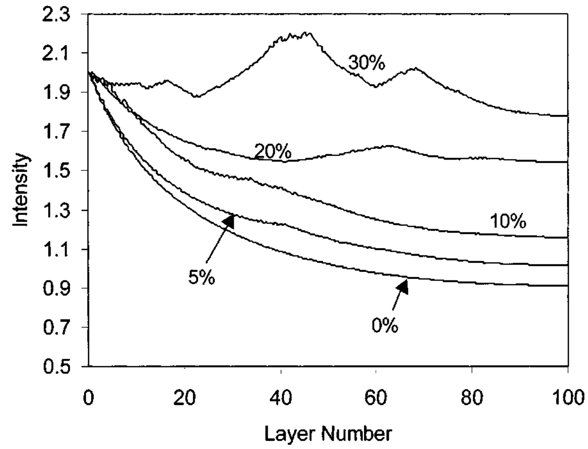


Fig. 9. Evolution of the intensity of the forward propagating wave across the structure. Impact of the increasing level of randomness.

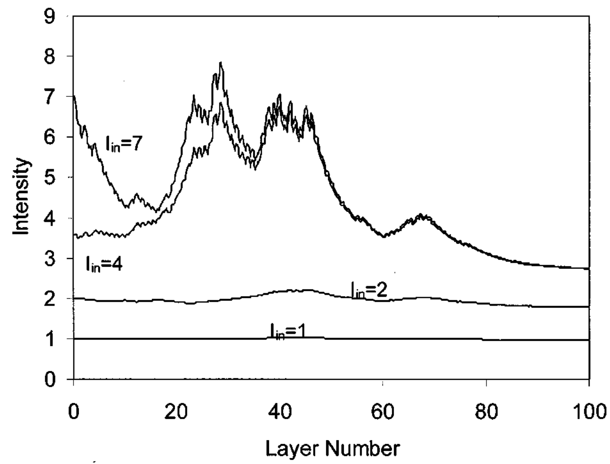


Fig. 10. Intensity of forward propagating wave across a structure consisting of five randomized 500-layer units. The insert shows transmittance spectra for one and five units.

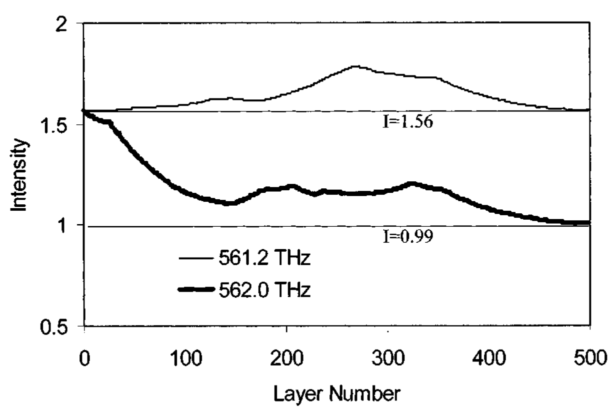


Fig. 11. Intensity across the structure in the transmitting and limiting regimes.

is evident from a comparison of the curves corresponding to $I_{\rm in} = 3.5$ and $I_{\rm in} = 7$, for high incident intensity the transmitted intensity is constant. This confirms the limiting behavior of these structures.

As discussed and demonstrated in Section III-A, introducing disorder may result in certain waves having high transmittance, regardless of whether these waves lie within the stopband of the original, ordered structure. We illustrate in Figs. 11 and 12 how the use of optical isolators eliminates this problem.

We show in Fig. 11 the evolution of the intensity associated with the forward-propagating wave in the limiting and transmitting states. We consider a 7%-randomized, 500-layer structure with $|n_{\rm nl}| = 0.002$. Within the limiting domain (562 THz), the intensity decays across the structure. When the transmittance maximum is reached (561.2 THz), a spatial soliton is formed.

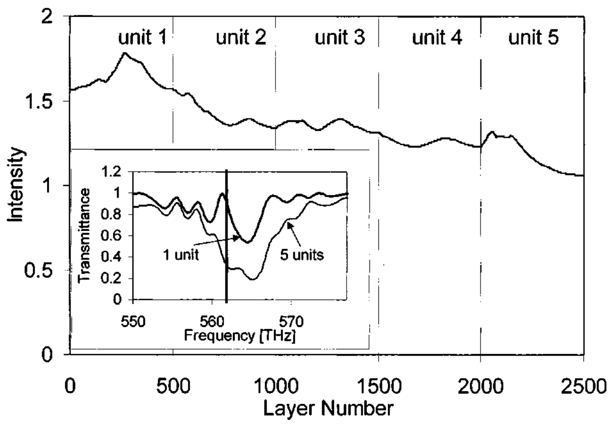


Fig. 12. Intensity of forward propagating wave across a structure consisting of five randomized 500-layer units. The insert shows transmittance spectra for one and five units.

We plot in Fig. 12 the evolution of the forward-propagating intensity across a structure consisting of four units separated by isolators. The degree of randomization and the structural and material parameters are the same as in Fig. 12. For incident intensity $I_{\text{in}} = 1.56$ and frequency 561.2 THz, light is fully transmitted by the first unit. On its own, this first segment does not provide limiting, but, instead, possesses a transmittance spike for this particular choice of incident wave. The combined system, however, serves as an excellent limiter in view of isolator-based removal of backward feedback.

IV. Conclusions

We have analyzed broad-band optical limiting in disordered nonlinear structures that are periodic on average. The structures exhibit localization of light over a spectral bandwidth related to both the incident intensity and the degree of disorder. We have shown that even highly disordered structures exhibit true optical limiting over a spectral range much greater than the limiting bandwidth of perfectly periodic nonlinear media.

Extension of the proposed one-dimensional (1-D) systems to three-dimensional (3-D) nonlinear periodic structures will yield limiters that provide full protection against incident radiation for an arbitrary intensity, angle of incidence, and optical wavelength. By opening up further spatial degrees of freedom, 3-D structures will add the element of intensity-tunable optical nonlinear diffraction to the 1-D transmission and reflection effects explored in this work. Future work will also include the application of analytical statistical methods based on the analysis of cyclostationarity to generalization of the properties of partially disordered nonlinear media.

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